Exercise N.5

Exercise

Evaluate the following integrals involving the Dirac delta function

$$a) \quad \int_{-3}^{1} dx \ (x^3 - 3x^2 + 2x - 1) \cdot \delta(x)$$

$$b) \quad \int_{0}^{\infty} dx \ [cos(3x) \ + 2] \cdot \delta(x - \pi)$$

$$c) \quad \int_{-1}^{1} dx \ e^x \cdot \delta(x - 2)$$

$$d) \quad \int_{-\infty}^{+\infty} dx \ e^{|x|-3} \cdot \delta(x + 2)$$

Solution

a)

$$\int_{-3}^{1} dx\ (x^3 - 3x^2 + 2x - 1) \cdot \delta(x)$$

Let us define

$$F(x) = x^3 - 3x^2 + 2x - 1$$

We have to evaluate

$$\int_{-3}^{1} dx\ F(x) \cdot \delta(x)$$

and, since $x = 0$ is inside the integration region $[-3, 1]$, this integral holds

$$F(0) = -1$$

Solution

b)

$$\int_0^\infty dx\ [cos(3x)\ + 2] \cdot \delta(x - \pi)$$

Let us define

$$F(x) = cos(3x)\ + 2$$

We have to evaluate

$$\int_0^\infty dx\ F(x) \cdot \delta(x - \pi)$$

and, since $x = \pi$ is inside the integration region $[0, +\infty]$, this integral holds

$$F(\pi) = cos(3\pi) + 2 = -1 + 2 = 1$$

Solution

c)

$$\int_{-1}^{1} dx\ e^x \cdot \delta(x-2)$$

Let us define

$$F(x) = e^x$$

We have to evaluate

$$\int_{-1}^{1} dx\ F(x) \cdot \delta(x-2)$$

and, since $x = 2$ is outside the integration region $[-1, +1]$, the integral is null.

Solution

d)

$$\int_{-\infty}^{+\infty} dx\ e^{|x|-3} \cdot \delta(x+2)$$

Let us define

$$F(x) = e^{|x|-3}$$

We have to evaluate

$$\int_{-\infty}^{+\infty} dx\ F(x) \cdot \delta(x+2)$$

and, since $x = -2$ is inside the integration region $[-\infty, +\infty]$, the integral holds

$$F(-2) = e^{|-2|-3} = e^{2-3} = e^{-1}$$