QUANTUM MECHANICS Lecture 1

Introduction

Enrico Iacopini

September 3, 2019

D. J. Griffiths: Preface

Few informations before to start ...

1. My name: Enrico Iacopini
2. e-mail adress: iacopini@unifi.it
3. Textbook used:
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   Author: David J. Griffiths
   ISBN-13 978 0131911758
4. My transparencies
   (your e-mail adresses ... please)
5. Field trip: visit to LENS laboratory in Florence November 15 (morning)

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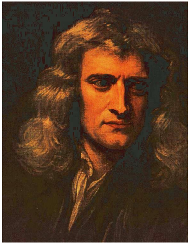

According to the second Newton's law, the dynamics of a point-like mass m, subject to a force $\vec{f}$, is described by the equation of motion

$$m\frac{d^2\vec{x}}{dt^2} \equiv m\,\ddot{\vec{x}} = \vec{f}(\vec{x},t)$$

1. By integrating this differential equation, one obtains the particle velocity $\dot{\vec{x}}(t)$ in terms of the *initial condition* $\dot{\vec{x}}(t=0) \equiv \dot{\vec{x}}_0$ and, after another integration, we obtain the particle trajectory $\vec{x}(t)$ in terms also of the initial position $\vec{x}(t=0) \equiv \vec{x}_0$.

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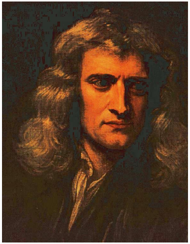

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If the force $\vec{f}$ is *conservative*, which means that it exists a function $V(\vec{x})$ such that

$$\vec{f}(\vec{x}) = -\vec{\nabla} V(\vec{x}) \equiv -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

then, by defining the kinetic energy T in terms of $\vec{v} \equiv \dot{\vec{x}}$

$$T \equiv \frac{1}{2} m \left|\vec{v}\right|^2$$

it turns out that the quantity $E = T + V$, which represents the **total mechanical energy**, remains constant along the time (we say that E *is a constant of motion*).

Introduction

If the energy potential is *central*, which means that

$$V(\vec{x}) = V(|\vec{x}|) \equiv V(r)$$

then there exists another *independent* constant of motion: the angular momentum $\vec{L}$

$$\begin{aligned}\vec{L} &\equiv \vec{x} \times (m\,\vec{v}) = \\ &= m\,(yv_z - zv_y, zv_x - xv_z, xv_y - yv_x)\end{aligned}$$

Introduction

Within the framework of Classical Mechanics, by integrating the Newton's second law differential equation and/or by making use of the constants of motion described above, we are able

1. to describe the free fall of an apple;
2. to determine the planets orbits;
3. to justify the pendulum isochronism;
4. to explain the reason of the tides;

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Introduction: why QM ?

At the beginning of XX century, **three** important **problems** were still **unexplained**

1. the result from the **Michelson-Morley experiment;**
2. the spectrum of the **black-body radiation;**
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Introduction: why QM ?

The solution of the first problem has been found in the theory of Special Relativity, which has led to a new Mechanics, able to describe the motion of particles with velocities near the speed of light.

But this is another story ...

Introduction: why QM ?

Quantum Mechanics was developed as a response to the inability of the classical theories of Mechanics and Electromagnetism to provide a satisfactory explanation of the atomic structure and of the properties of electromagnetic radiation (black body).

Introduction: why QM ?

1. It is well known that an accelerated charge radiates electromagnetic energy.
   From the Larmor formula, the radiated power is given by

$$\mathcal{P} \equiv \frac{dE}{dt} = \frac{2}{3}\,\frac{e^2}{c^3}\,|\vec{a}|^2$$

2. But, then, why the electrons, orbiting around the nucleus (Rutherford model), **do not loose energy** and collapse on it ?

3. Using the Larmor formula, the collapsing time τ for of an hydrogen atom, assuming to start from an electron on a circular orbit of radius $R = Bohr\ radius \approx 0.5 \cdot 10^{-10} m$, is $\tau \approx 10^{-11} s$ ...

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The first attempt to explain the (hydrogen) atom stability is due to Niels Bohr (1913).

He simply **assumed** that the electron orbit is circular and that it does not radiate if its angular momentum is an integer multiple of $\hbar \equiv \frac{h}{2\pi}$, where

$h = 6.626 \times 10^{-34} J \cdot s$

is the Planck constant, previously introduced to explain the black body radiation spectrum.

According to the model, the radiation is emitted only when the electron changes its orbit and the light frequency is $\nu = \frac{\Delta E}{h}$, where ΔE is the energy difference between the two orbits.

Introduction: why QM ?

1. Let us see, in more detail, the consequences of Bohr's hypothesis.
2. From Newton's second law

$$F = \frac{e^2}{4\pi\epsilon_0 r^2} = m\, r\, \omega^2 \Rightarrow$$

$$\Rightarrow V(r) = -\frac{e^2}{4\pi\epsilon_0 r} = -m\, r^2\, \omega^2 = -2T$$

3. Therefore, the electron total energy reads

$$E = T + V = T - 2T = -T = \frac{1}{2}V =$$

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1. If we impose, now, the Bohr quantization condition $L = n\hbar$, we have that not all the values for the radius r are possible, but only those for which

$$r_n = \frac{4\pi\epsilon_0}{me^2}\,(n\hbar)^2$$

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$$R_H \equiv \frac{m}{2}\left(\frac{e^2}{4\pi\epsilon_0\hbar}\right)^2 \frac{1}{2\pi\hbar c} \approx 109,677.6\, cm^{-1}$$

become

$$E_n = -R_H \frac{2\pi\hbar c}{n^2} \equiv -R_H \frac{hc}{n^2}$$

2. With this formula, Bohr was able to explain the experimental hydrogen spectrum:
3. Lyman lines: $\frac{1}{\lambda_m} = R_H(1 - \frac{1}{m^2})$, $m = 2, 3, \ldots$
4. Balmer lines: $\frac{1}{\lambda_k} = R_H(\frac{1}{4} - \frac{1}{k^2})$, $k = 3, 4, \ldots$

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Introduction: why QM ?

A better understanding of the situation was reached when De Broglie proposed its hypothesis of the **wave-particle duality**.

Introduction: why QM ?

1. In the same way as to explain the Compton effect (scattering electron-photon in which the photon changes its frequency) one needs to assume the **corpuscolar** nature of the electromagnetic radiation (photon)

2. to explain the hydrogen atom spectrum (Bohr model) we have to assume the **ondulatory** nature of the electron, by assigning to it a wavelength $\lambda = \frac{\mathbf{h}}{\mathbf{mv}} \equiv \frac{\mathbf{h}}{\mathbf{p}}$ where $\mathbf{p} \equiv \mathbf{m\,v}$ is the modulus of its linear momentum.

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1. With this hypothesis, the Bohr condition was simply that the wave associated to the orbiting electron had to be **a stationary wave**, in fact

$$2\pi r_n = n\lambda = n\frac{h}{mv} \Rightarrow L = mvr_n = n\frac{h}{2\pi} \equiv n\hbar$$

2. With the De Broglie hypothesis, the **dualism wave – particle** has become a fundamental ingredient of QM, although not intuitive and, for sure, **not easy to understand !**

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1. It took quite a long time (until 1927) to arrive to a formalism able to describe what happens in the microscopic world, together with a coherent interpretation of the mathematical formalism behind it (not yet without problems...).
2. The point is that **Quantum Mechanics represents a revolutionary departure** from classical ideas and in many aspects it appears to be counterintuitive.
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